$$\frac{dP}{dx} = \frac{V\beta_f \frac{dT}{dx} + (V_2 - V_1) \frac{df}{dx}}{VK_{T,f} - \frac{1}{m^2}} , \tag{D.13}$$

$$\frac{dT}{dx} = \frac{\frac{dP}{dx}\left(\frac{P}{m^2} - PVK_{T,f} + TV\beta_f\right) - (E_2 - E_1) \frac{df}{dx}}{C_{P,f} - PV\beta_f} . \tag{D.14}$$

Equations (D.13) and (D.14) require only an equation of state and a relation for df/dx to solve for the stress and temperature distribution in a steady wave.

For iron the second term on the right hand side of Eq. (D.13) is about 60 times greater than the first term. Steady profile calculations with this first term neglected (for example see Fig. 6.2) are called temperature independent.

D.2. Rate Equation

Horie and Duvall[20] assumed a relation for df/dt (see Chapter 6) that for steady waves can be written as

$$U \frac{df}{dx} = \frac{f^{eq} - f}{\tau_1} . \tag{D.15}$$

Andrews[27,29] later showed that by defining τ as the constant energy and constant volume equilibration time, the rate equation can be directly related to the difference in Gibbs energies. For steady waves this rate equation becomes:

$$U \frac{df}{dx} = - \frac{J\ G_{21}}{|A|\ \tau_2} . \tag{D.16}$$

For small variations from equilibrium the determinant of A is

$$|A| = \begin{vmatrix} -VK_{T,f} & \beta_f V & \Delta V \\ PVK_{T,f} - TV\beta_f & C_{P,f} - PV\beta_f & \Delta E \\ \Delta V & -\Delta S & 0 \end{vmatrix}$$

$$= \Delta E(\beta V\Delta V - VK_T\Delta S) + \Delta V\Delta S(TV\beta - PVK_T) + (\Delta V)^2(PV\beta - C_P) \tag{D.17}$$

where $\Delta S = S_2 - S_1$ and $\Delta V = V_2 - V_1$.

The transformation Jacobian J was previously evaluated by Hayes[16] as

$$J = J\begin{pmatrix} VEf \\ PTf \end{pmatrix} = -C_{P,f}VK_{S,f} \ . \tag{D.18}$$

Equation (D.16) provides the necessary relation defining df/dx. Equations (D.13), (D.14), (D.17), and (D.18), and an equation of state give the stress and temperature distribution in the steady shock.